Paper Reference(s)

6668/01

Edexcel GCE

Further Pure Mathematics FP2

Advanced

Friday 19 June 2009 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Orange)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Further Pure Mathematics FP2), the paper reference (6668), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

1. (*a*) Express $\frac{1}{r(r+2)}$ in partial fractions.

(1)

(*b*) Hence show that $\sum_{r=1}^{n} \frac{4}{r(r+2)} = \frac{n(3n+5)}{(n+1)(n+2)}$.

(5)

---

2. Solve the equation

$$z^3 = 4\sqrt{2} - 4\sqrt{2}\text{i},$$

giving your answers in the form $r(\cos\theta + \text{i}\sin\theta)$, where $-\pi < \theta \leq \pi$.

(6)

---

3. Find the general solution of the differential equation

$$\sin x\frac{\text{d}y}{\text{d}x} - y\cos x = \sin 2x \sin x$$

giving your answer in the form $y = \text{f}(x)$.

(8)

4.

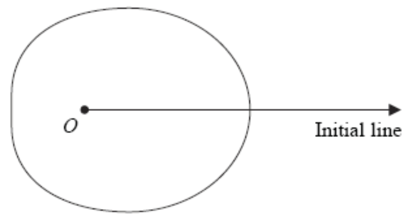

Figure 1

Figure 1 shows a sketch of the curve with polar equation

$$r = a + 3\cos\theta, \quad a > 0, \quad 0 \leq \theta < 2\pi.$$

The area enclosed by the curve is $\frac{107}{2}\pi$.

Find the value of a.

(8)

---

5. $$y = \sec^2 x$$

(*a*) Show that $\dfrac{d^2y}{dx^2} = 6\sec^4 x - 4\sec^2 x$.

(4)

(*b*) Find a Taylor series expansion of $\sec^2 x$ in ascending powers of $\left(x - \dfrac{\pi}{4}\right)$, up to and including the term in $\left(x - \dfrac{\pi}{4}\right)^3$.

(6)

---

6. A transformation T from the z-plane to the w-plane is given by

$$w = \frac{z}{z + i}, \quad z \neq -i.$$

The circle with equation $|z| = 3$ is mapped by T onto the curve C.

(*a*) Show that C is a circle and find its centre and radius.

(8)

The region $|z| < 3$ in the z-plane is mapped by T onto the region R in the w-plane.

(*b*) Shade the region R on an Argand diagram.

(2)

---

7. (*a*) Sketch the graph of $y = |x^2 - a^2|$, where $a > 1$, showing the coordinates of the points where the graph meets the axes.

(2)

(*b*) Solve $|x^2 - a^2| = a^2 - x$, $a > 1$.

(6)

(*c*) Find the set of values of x for which $|x^2 - a^2| > a^2 - x$, $a > 1$.

(4)

---

8. $$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 2e^{-t}.$$

Given that $x = 0$ and $\dfrac{dx}{dt} = 2$ at $t = 0$,

(*a*) find x in terms of t.

(8)

The solution to part (*a*) is used to represent the motion of a particle P on the x-axis. At time t seconds, where $t > 0$, P is x metres from the origin O.

(*b*) Show that the maximum distance between O and P is $\dfrac{2\sqrt{3}}{9}$ m and justify that this distance is a maximum.

(7)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6668/01

Edexcel GCE

Further Pure Mathematics FP2

Advanced Subsidiary

Thursday 24 June 2010 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Further Pure Mathematics FP2), the paper reference (6668), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

N35388A

1. (*a*) Express $\dfrac{3}{(3r-1)(3r+2)}$ in partial fractions.

(2)

(*b*) Using your answer to part (a) and the method of differences, show that

$$\sum_{r=1}^{n} \frac{3}{(3r-1)(3r+2)} = \boxed{}.$$

(3)

(*c*) Evaluate $\displaystyle\sum_{r=100}^{1000} \frac{3}{(3r-1)(3r+2)}$, giving your answer to 3 significant figures.

(2)

---

2. The displacement x metres of a particle at time t seconds is given by the differential equation

$$\frac{d^2x}{dt^2} + x + \cos x = 0.$$

When $t = 0$, $x = 0$ and $\dfrac{dx}{dt} = \dfrac{1}{2}$.

Find a Taylor series solution for x in ascending powers of t, up to and including the term in t^3.

(5)

---

3. (*a*) Find the set of values of x for which

$$x + 4 > \frac{2}{x+3}.$$

(6)

(*b*) Deduce, or otherwise find, the values of x for which

$$x + 4 > \frac{2}{|x+3|}.$$

(1)

---

4. $$z = -8 + (8\sqrt{3})\text{i}$$

(*a*) Find the modulus of z and the argument of z. **(3)**

Using de Moivre's theorem,

(*b*) find z^3, **(2)**

(*c*) find the values of w such that $w^4 = z$, giving your answers in the form $a + \text{i}b$, where $a, b \in \mathbb{R}$. **(5)**

5.

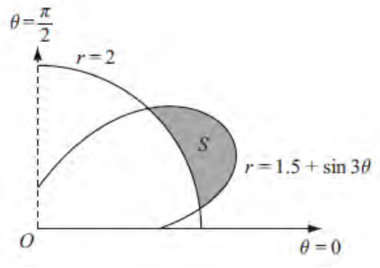

Figure 1

Figure 1 shows the curves given by the polar equations

$$r = 2, \qquad 0 \le \theta \le \frac{\pi}{2},$$

$$\text{and} \quad r = 1.5 + \sin 3\theta, \qquad 0 \le \theta \le \frac{\pi}{2}.$$

(*a*) Find the coordinates of the points where the curves intersect. **(3)**

The region S, between the curves, for which $r > 2$ and for which $r < (1.5 + \sin 3\theta)$, is shown shaded in Figure 1.

(*b*) Find, by integration, the area of the shaded region S, giving your answer in the form $a\pi + b\sqrt{3}$, where a and b are simplified fractions. **(7)**

---

6. A complex number z is represented by the point P in the Argand diagram.

(*a*) Given that $|z - 6| = |z|$, sketch the locus of P. **(2)**

(*b*) Find the complex numbers z which satisfy both $|z - 6| = |z|$ and $|z - 3 - 4\text{i}| = 5$. **(3)**

The transformation T from the z-plane to the w-plane is given by $w = \dfrac{30}{z}$.

(*c*) Show that T maps $|z - 6| = |z|$ onto a circle in the w-plane and give the cartesian equation of this circle. **(5)**

---

7. (*a*) Show that the transformation $z = y^{\frac{1}{2}}$ transforms the differential equation

$$\frac{\text{d}y}{\text{d}x} - 4y\tan x = 2y^{\frac{1}{2}} \qquad \text{(I)}$$

into the differential equation

$$\frac{\text{d}z}{\text{d}x} - 2z\tan x = 1 \qquad \text{(II)}$$

(5)

(*b*) Solve the differential equation (II) to find z as a function of x. **(6)**

(*c*) Hence obtain the general solution of the differential equation (I). **(1)**

---

8. (*a*) Find the value of λ for which $y = \lambda x \sin 5x$ is a particular integral of the differential equation

$$\frac{d^2y}{dx^2} + 25y = 3\cos 5x.$$

(4)

(*b*) Using your answer to part (a), find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 25y = 3\cos 5x.$$

(3)

Given that at $x = 0$, $y = 0$ and $\frac{dy}{dx} = 5$,

(*c*) find the particular solution of this differential equation, giving your solution in the form $y = f(x)$.

(5)

(*d*) Sketch the curve with equation $y = f(x)$ for $0 \leq x \leq \pi$.

(2)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6668/01

Edexcel GCE

Further Pure Mathematics FP2

Advanced Level

Thursday 23 June 2011 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics FP2), the paper reference (6668), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P38168A

1. Find the set of values of x for which

$$\frac{3}{x+3} > \frac{x-4}{x}.$$

(7)

2.
$$\frac{d^2y}{dx^2} = e^x\left(2y\frac{dy}{dx} + y^2 + 1\right).$$

(*a*) Show that

$$\frac{d^3y}{dx^3} = e^x\left[2y\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 + ky\frac{dy}{dx} + y^2 + 1\right],$$

where k is a constant to be found.

(3)

Given that, at $x = 0$, $y = 1$ and $\frac{dy}{dx} = 2$,

(*b*) find a series solution for y in ascending powers of x, up to and including the term in x^3.

(4)

---

3. Find the general solution of the differential equation

$$x\frac{dy}{dx} + 5y = \frac{\ln x}{x}, \quad x > 0,$$

giving your answer in the form $y = f(x)$.

(8)

---

4. Given that

$$(2r + 1)^3 = Ar^3 + Br^2 + Cr + 1,$$

(*a*) find the values of the constants A, B and C.

(2)

(*b*) Show that

$$(2r + 1)^3 - (2r - 1)^3 = 24r^2 + 2.$$

(2)

(*c*) Using the result in part (*b*) and the method of differences, show that

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n + 1)(2n + 1).$$

(5)

---

5. The point P represents the complex number z on an Argand diagram, where

$$|z - i| = 2.$$

The locus of P as z varies is the curve C.

(*a*) Find a cartesian equation of C.

(2)

(*b*) Sketch the curve C.

(2)

A transformation T from the z-plane to the w-plane is given by

$$w = \frac{z + i}{3 + iz}, \quad z \neq 3i.$$

The point Q is mapped by T onto the point R. Given that R lies on the real axis,

(*c*) show that Q lies on C.

(5)

6.

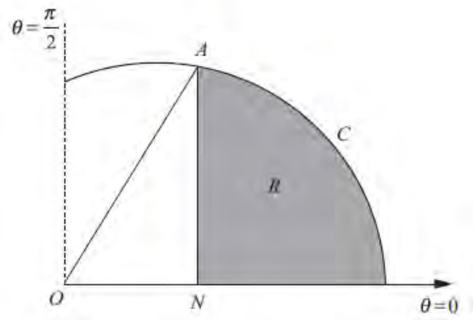

Figure 1

The curve C shown in Figure 1 has polar equation

$$r = 2 + \cos\theta, \quad 0 \leq \theta \leq \frac{\pi}{2}.$$

At the point A on C, the value of r is $\frac{5}{2}$.

The point N lies on the initial line and AN is perpendicular to the initial line.

The finite region R, shown shaded in Figure 1, is bounded by the curve C, the initial line and the line AN.

Find the exact area of the shaded region R.

(9)

---

7. (*a*) Use de Moivre's theorem to show that

$$\sin 5\theta = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta.$$

(5)

Hence, given also that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$,

(*b*) find all the solutions of

$$\sin 5\theta = 5\sin 3\theta,$$

in the interval $0 \leq \theta < 2\pi$. Give your answers to 3 decimal places.

(6)

---

8. The differential equation

$$\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 9x = \cos 3t, \qquad t \geq 0,$$

describes the motion of a particle along the x-axis.

(*a*) Find the general solution of this differential equation.

(8)

(*b*) Find the particular solution of this differential equation for which, at $t = 0$, $x = \frac{1}{2}$ and $\frac{dx}{dt} = 0$.

(5)

On the graph of the particular solution defined in part (*b*), the first turning point for $T > 30$ is the point A.

(*c*) Find approximate values for the coordinates of A.

(2)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6668/01

Edexcel GCE

Further Pure Mathematics FP2

Advanced Level

Friday 22 June 2012 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics FP2), the paper reference (6668), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P40104A

1. Find the set of values of x for which

$$|x^2 - 4| > 3x.$$

(5)

---

2. The curve C has polar equation

$$r = 1 + 2\cos\theta, \quad 0 \leq \theta \leq \frac{\pi}{2}.$$

At the point P on C, the tangent to C is parallel to the initial line.

Given that O is the pole, find the exact length of the line OP.

(7)

---

3. (*a*) Express the complex number $-2 + (2\sqrt{3})\mathrm{i}$ in the form $r(\cos\theta + \mathrm{i}\sin\theta)$, $-\pi < \theta \leq \pi$.

(3)

(*b*) Solve the equation

$$z^4 = -2 + (2\sqrt{3})\mathrm{i}$$

giving the roots in the form $r(\cos\theta + \mathrm{i}\sin\theta)$, $-\pi < \theta \leq \pi$.

(5)

---

4. Find the general solution of the differential equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 5\frac{\mathrm{d}x}{\mathrm{d}t} + 6x = 2\cos t - \sin t.$$

(9)

5.

$$x\frac{\mathrm{d}y}{\mathrm{d}x} = 3x + y^2.$$

(*a*) Show that

$$x\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + (1 - 2y)\frac{\mathrm{d}y}{\mathrm{d}x} = 3.$$

(2)

Given that $y = 1$ at $x = 1$,

(*b*) find a series solution for y in ascending powers of $(x - 1)$, up to and including the term in $(x - 1)^3$.

(8)

---

6. (*a*) Express $\dfrac{1}{r(r+2)}$ in partial fractions.

(2)

(*b*) Hence prove, by the method of differences, that

$$\sum_{r=1}^{n} \frac{1}{r(r+2)} = \frac{n(an+b)}{4(n+1)(n+2)},$$

where a and b are constants to be found.

(6)

(*c*) Hence show that

$$\sum_{r=n+1}^{2n} \frac{1}{r(r+2)} = \frac{n(4n+5)}{4(n+1)(n+2)(2n+1)}.$$

(3)

---

7. (*a*) Show that the substitution $y = vx$ transforms the differential equation

$$3xy^2 \frac{\mathrm{d}y}{\mathrm{d}x} = x^3 + y^3 \qquad \text{(I)}$$

into the differential equation

$$3v^2 x \frac{\mathrm{d}v}{\mathrm{d}x} = 1 - 2v^3 \qquad \text{(II)}$$

(3)

(*b*) By solving differential equation (II), find a general solution of differential equation (I) in the form $y = \mathrm{f}(x)$.

(6)

Given that $y = 2$ at $x = 1$,

(*c*) find the value of $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ at $x = 1$.

(2)

---

8. The point P represents a complex number z on an Argand diagram such that

$$|z - 6\mathrm{i}| = 2|z - 3|.$$

(*a*) Show that, as z varies, the locus of P is a circle, stating the radius and the coordinates of the centre of this circle.

(6)

The point Q represents a complex number z on an Argand diagram such that

$$\arg(z - 6) = -\frac{3\pi}{4}.$$

(*b*) Sketch, on the same Argand diagram, the locus of P and the locus of Q as z varies.

(4)

(*c*) Find the complex number for which both $|z - 6\mathrm{i}| = 2|z - 3|$ and $\arg(z - 6) = -\dfrac{3\pi}{4}$.

(4)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6668/01R

Edexcel GCE

Further Pure Mathematics FP2 (R)

Advanced/Advanced Subsidiary

Friday 21 June 2013 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 32 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P42955A

1. A transformation T from the z-plane to the w-plane is given by

$$w = \frac{z + 2\text{i}}{\text{i}z} \qquad z \neq 0$$

The transformation maps points on the real axis in the z-plane onto a line in the w-plane.

Find an equation of this line.

(4)

---

2. Use algebra to find the set of values of x for which

$$\frac{6x}{3 - x} > \frac{1}{x + 1}$$

(7)

---

3. (*a*) Express $\dfrac{2}{(r+1)(r+3)}$ in partial fractions.

(2)

(*b*) Hence show that

$$\sum_{r=1}^{n} \frac{2}{(r+1)(r+3)} = \frac{n(5n+13)}{6(n+2)(n+3)}$$

(4)

(*c*) Evaluate $\displaystyle\sum_{r=10}^{100} \frac{2}{(r+1)(r+3)}$, giving your answer to 3 significant figures.

(2)

---

4. Given that

$$y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2+5y=0$$

(*a*) find $\frac{d^3y}{dx^3}$ in terms of $\frac{d^2y}{dx^2}$, $\frac{dy}{dx}$ and y.

(4)

Given that $y = 2$ and $\frac{dy}{dx} = 2$ at $x = 0$

(*b*) find a series solution for y in ascending powers of x, up to and including the term in x^3.

(5)

---

5. (*a*) Find, in the form $y = f(x)$, the general solution of the equation

$$\frac{dy}{dx}+2y\tan x=\sin 2x, \qquad 0 < x < \frac{\pi}{2}$$

(6)

Given that $y = 2$ at $x = \frac{\pi}{3}$

(*b*) find the value of y at $x = \frac{\pi}{6}$, giving your answer in the form $a + k \ln b$, where a and b are integers and k is rational.

(4)

---

6. The complex number $z = e^{i\theta}$, where θ is real.

(*a*) Use de Moivre's theorem to show that

$$z^n + \frac{1}{z^n} = 2\cos n\theta$$

where n is a positive integer.

(2)

(*b*) Show that

$$\cos^5\theta = \frac{1}{16}(\cos 5\theta + 5\cos 3\theta + 10\cos\theta)$$

(5)

(*c*) Hence find all the solutions of

$$\cos 5\theta + 5\cos 3\theta + 12\cos\theta = 0$$

in the interval $0 \leq \theta < 2\pi$.

(4)

---

7. (*a*) Find the value of λ for which $\lambda t^2 e^{3t}$ is a particular integral of the differential equation

$$\frac{d^2y}{dt^2}-6\frac{dy}{dt}+9y=6e^{3t}, \qquad t \geq 0$$

(5)

(*b*) Hence find the general solution of this differential equation.

(3)

Given that when $t = 0$, $y = 5$ and $\frac{dy}{dt} = 4$

(*c*) find the particular solution of this differential equation, giving your solution in the form $y = f(t)$.

(5)

8.

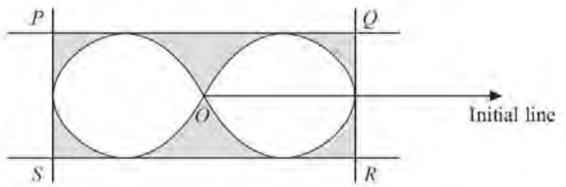

Figure 1

Figure 1 shows a closed curve C with equation

$$r = 3(\cos 2\theta)^{\frac{1}{2}}, \qquad \text{where } -\frac{\pi}{4} < \theta \leq \frac{\pi}{4},\ \frac{3\pi}{4} < \theta \leq \frac{5\pi}{4}$$

The lines PQ, SR, PS and QR are tangents to C, where PQ and SR are parallel to the initial line and PS and QR are perpendicular to the initial line. The point O is the pole.

(*a*) Find the total area enclosed by the curve C, shown unshaded inside the rectangle in Figure 1.

(4)

(*b*) Find the total area of the region bounded by the curve C and the four tangents, shown shaded in Figure 1.

(9)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6668/01

Edexcel GCE

Further Pure Mathematics FP2

Advanced/Advanced Subsidiary

Friday 21 June 2013 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P43149A

1. (*a*) Express $\dfrac{2}{(2r+1)(2r+3)}$ in partial fractions.

(2)

(*b*) Using your answer to (*a*), find, in terms of *n*,

$$\sum_{r=1}^{n} \frac{3}{(2r+1)(2r+3)}$$

Give your answer as a single fraction in its simplest form.

(3)

2.

$$z = 5\sqrt{3} - 5\text{i}$$

Find

(*a*) $|z|$,

(1)

(*b*) $\arg(z)$, in terms of π.

(2)

$$w = 2\left(\cos\frac{\pi}{4} + \text{i}\sin\frac{\pi}{4}\right)$$

Find

(*c*) $\left|\dfrac{w}{z}\right|$,

(1)

(*d*) $\arg\left|\dfrac{w}{z}\right|$, in terms of π.

(2)

3.

$$\frac{d^2y}{dx^2} + 4y - \sin x = 0$$

Given that $y = \dfrac{1}{2}$ and $\dfrac{dy}{dx} = \dfrac{1}{8}$ at $x = 0$,
find a series expansion for y in terms of x, up to and including the term in x^3.

(5)

---

4. (*a*) Given that

$$z = r(\cos\theta + \text{i}\sin\theta), \qquad r \in \mathbb{R}$$

prove, by induction, that $z^n = r^n(\cos n\theta + \text{i}\sin n\theta)$, $\quad n \in \mathbb{Z}^+$.

(5)

$$w = 3\left(\cos\frac{3\pi}{4} + \text{i}\sin\frac{3\pi}{4}\right)$$

(*b*) Find the exact value of w^5, giving your answer in the form $a + \text{i}b$, where $a, b \in \mathbb{R}$.

(2)

---

5. (*a*) Find the general solution of the differential equation

$$x\frac{dy}{dx} + 2y = 4x^2$$

(5)

(*b*) Find the particular solution for which $y = 5$ at $x = 1$, giving your answer in the form $y = \text{f}(x)$.

(2)

(*c*) (i) Find the exact values of the coordinates of the turning points of the curve with equation $y = \text{f}(x)$, making your method clear.

(ii) Sketch the curve with equation $y = \text{f}(x)$, showing the coordinates of the turning points.

(5)

---

6. (*a*) Use algebra to find the exact solutions of the equation

$$|2x^2 + 6x - 5| = 5 - 2x$$

(6)

(*b*) On the same diagram, sketch the curve with equation $y = |2x^2 + 6x - 5|$ and the line with equation $y = 5 - 2x$, showing the x-coordinates of the points where the line crosses the curve.

(3)

(*c*) Find the set of values of x for which

$$|2x^2 + 6x - 5| > 5 - 2x$$

(3)

---

7. (*a*) Show that the transformation $y = xv$ transforms the equation

$$4x^2\frac{d^2y}{dx^2} - 8x\frac{dy}{dx} + (8 + 4x^2)y = x^4 \qquad \text{(I)}$$

into the equation

$$4\frac{d^2v}{dx^2} + 4v = x \qquad \text{(II)}$$

(6)

(*b*) Solve the differential equation (II) to find v as a function of x.

(6)

(*c*) Hence state the general solution of the differential equation (I).

(1)

8.

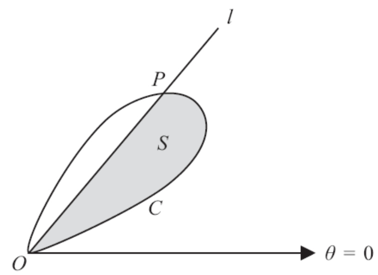

Figure 1

Figure 1 shows a curve C with polar equation $r = a\sin 2\theta$, $0 \leq \theta \leq \frac{\pi}{2}$, and a half-line l.

The half-line l meets C at the pole O and at the point P. The tangent to C at P is parallel to the initial line. The polar coordinates of P are (R, φ).

(*a*) Show that $\cos\varphi = \frac{1}{\sqrt{3}}$.

(6)

(*b*) Find the exact value of R.

(2)

The region S, shown shaded in Figure 1, is bounded by C and l.

(*c*) Use calculus to show that the exact area of S is

$$\frac{1}{36}a^2\left(9\arccos\left(\frac{1}{\sqrt{3}}\right) + \sqrt{2}\right)$$

(7)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

WFM02/01

Pearson Edexcel International Advanced Level

Further Pure Mathematics F2

Advanced/Advanced Subsidiary

Friday 6 June 2014 – Afternoon

Time: 1 hour 30 minutes

<u>**Materials required for examination**</u>	<u>**Items included with question papers**</u>
Mathematical Formulae (Blue)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

P44517A

1. (*a*) Show that

$$\frac{1}{(r+1)(r+2)(r+3)} \equiv \frac{1}{2(r+1)(r+2)} - \frac{1}{2(r+2)(r+3)}$$

(2)

(*b*) Hence, or otherwise, find

$$\sum_{r=1}^{n} \frac{1}{(r+1)(r+2)(r+3)}$$

giving your answer as a single fraction in its simplest form.

(4)

---

2. Use algebra to find the set of values of x for which

$$\frac{6}{x-3} \leq x+2$$

(7)

---

3. Solve the equation

$$z^5 = 16 - 16\mathrm{i}\sqrt{3}$$

giving your answers in the form $r\mathrm{e}^{\mathrm{i}\theta}$ where θ is in terms of π and $0 \leq \theta < 2\pi$.

(5)

---

4. A transformation from the z-plane to the w-plane is given by

$$w = \frac{z}{z+3}, \qquad z \neq -3$$

Under this transformation, the circle $|z| = 2$ in the z-plane is mapped onto a circle C in the w-plane.

Determine the centre and the radius of the circle C.

(7)

---

5. $$\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$$

(*a*) Show that

$$\frac{d^4y}{dx^4} = \left(ax^2 + b\right)\frac{d^2y}{dx^2}$$

where *a* and *b* are constants to be found.

(5)

Given that $y = 1$ and $\frac{dy}{dx} = 3$ at $x = 0$,

(*b*) find a series solution for *y* in ascending powers of *x* up to and including the term in x^4.

(5)

(*c*) use your series to estimate the value of *y* at $x = -0.2$, giving your answer to four decimal places.

(2)

---

6. $$x\frac{dy}{dx} + (1-2x)y = x, \qquad x > 0$$

Find the general solution of the differential equation, giving your answer in the form $y = f(x)$.

(9)

---

7. The point *P* represents a complex number *z* on an Argand diagram, where

$$|z + 1| = |2z - 1|$$

and the point *Q* represents a complex number *w* on the Argand diagram, where

$$|w| = |w - 1 + i|$$

Find the exact coordinates of the points where the locus of *P* intersects the locus of *Q*.

(7)

---

8. (*a*) Show that the substitution $x = e^t$ transforms the differential equation

$$x^2\frac{d^2y}{dx^2} + 5x\frac{dy}{dx} + 13y = 0, \qquad x > 0 \quad \text{(I)}$$

into the differential equation

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 13y = 0$$

(7)

(*b*) Hence find the general solution of the differential equation (I).

(5)

9.

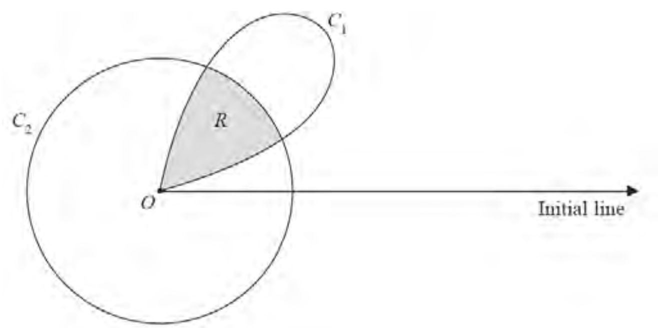

Figure 1

Figure 1 shows the curve C_1 with polar equation $r = 2a\sin 2\theta$, $0 \leq \theta \leq \frac{\pi}{2}$, and the circle C_2 with polar equation $r = a$, $0 \leq \theta \leq 2\pi$, where *a* is a positive constant.

(*a*) Find, in terms of *a*, the polar coordinates of the points where the curve C_1 meets the circle C_2.

(3)

The regions enclosed by the curve C_1 and the circle C_2 overlap and the common region *R* is shaded in Figure 1.

(*b*) Find the area of the shaded region *R*, giving your answer in the form $\frac{1}{12}a^2\left(p\pi + q\sqrt{3}\right)$, where *p* and *q* are integers to be found.

(7)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6668/01R

Edexcel GCE

Further Pure Mathematics FP2 (R)

Advanced/Advanced Subsidiary

Friday 6 June 2014 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P43150A

1. (*a*) Express $\frac{2}{4r^2-1}$ in partial fractions.

(2)

(*b*) Hence use the method of differences to show that

$$\sum_{r=1}^{n}\frac{2}{4r^2-1}=\frac{n}{2n+1}$$

(3)

---

2. Using algebra, find the set of values of x for which

$$3x-5<\frac{2}{x}$$

(5)

---

3. (*a*) Find the general solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x}+2y\tan x=\mathrm{e}^{4x}\cos^2 x, \qquad -\frac{\pi}{2}<x<\frac{\pi}{2}$$

giving your answer in the form $y = \mathrm{f}(x)$.

(6)

(*b*) Find the particular solution for which $y = 1$ at $x = 0$.

(2)

4.

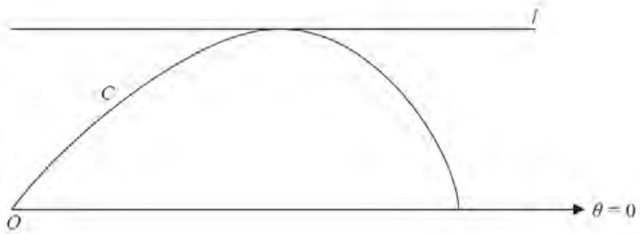

Figure 1

Figure 1 shows the curve C with polar equation

$$r = 2\cos 2\theta, \quad 0 \leq \theta \leq \frac{\pi}{4}$$

The line l is parallel to the initial line and is a tangent to C.

Find an equation of l, giving your answer in the form $r = \mathrm{f}(\theta)$.

(9)

5.

$$y\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 + 2y = 0$$

(*a*) Find an expression for $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$ in terms of $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$, $\frac{\mathrm{d}y}{\mathrm{d}x}$ and y.

(4)

Given that $y = 2$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = 0.5$ at $x = 0$,

(*b*) find a series solution for y in ascending powers of x, up to and including the term in x^3.

(5)

---

6. The transformation T maps points from the z-plane, where $z = x + \mathrm{i}y$, to the w-plane, where $w = u + \mathrm{i}v$.

The transformation T is given by

$$w = \frac{z}{\mathrm{i}z + 1}, \qquad z \neq \mathrm{i}$$

The transformation T maps the line l in the z-plane onto the line with equation $v = -1$ in the w-plane.

(*a*) Find a cartesian equation of l in terms of x and y.

(5)

The transformation T maps the line with equation $y = \frac{1}{2}$ in the z-plane onto the curve C in the w-plane.

(*b*) (i) Show that C is a circle with centre the origin.

(ii) Write down a cartesian equation of C in terms of u and v.

(6)

---

7. (*a*) Use de Moivre's theorem to show that

$$\sin 5\theta \equiv 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$$

(5)

(*b*) Hence find the five distinct solutions of the equation

$$16x^5 - 20x^3 + 5x + \frac{1}{2} = 0$$

giving your answers to 3 decimal places where necessary.

(5)

(*c*) Use the identity given in (*a*) to find

$$\int_0^{\frac{\pi}{4}} \left(4\sin^5\theta - 5\sin^3\theta\right)\mathrm{d}\theta$$

expressing your answer in the form $a\sqrt{2} + b$, where a and b are rational numbers.

(4)

---

8. (*a*) Show that the substitution $x = e^z$ transforms the differential equation

$$x^2 \frac{d^2 y}{dx^2} + 2x\frac{dy}{dx} - 2y = 3\ln x, \qquad x > 0 \qquad \text{(I)}$$

into the equation

$$\frac{d^2 y}{dz^2} + \frac{dy}{dz} - 2y = 3z \qquad \text{(II)}$$

(7)

(*b*) Find the general solution of the differential equation (II).

(6)

(*c*) Hence obtain the general solution of the differential equation (I) giving your answer in the form $y = f(x)$.

(1)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)
6668/01

Edexcel GCE

Further Pure Mathematics FP2

Advanced/Advanced Subsidiary

Friday 6 June 2014 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P44512A

1. (*a*) Express $\dfrac{2}{(r+2)(r+4)}$ in partial fractions.

(1)

(*b*) Hence show that

$$\sum_{r=1}^{n} \frac{2}{(r+2)(r+4)} = \frac{n(7n+25)}{12(n+3)(n+4)}$$

(5)

---

2. Use algebra to find the set of values of x for which

$$\left|3x^2 - 19x + 20\right| < 2x + 2$$

(6)

---

3. $$y = \sqrt{8 + e^x}, \qquad x \in \mathbb{R}$$

Find the series expansion for y in ascending powers of x, up to and including the term in x^2, giving each coefficient in its simplest form.

(8)

---

4. (*a*) Use de Moivre's theorem to show that

$$\cos 6\theta = 32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1$$

(5)

(*b*) Hence solve for $0 \leq \theta \leq \dfrac{\pi}{2}$

$$64\cos^6\theta - 96\cos^4\theta + 36\cos^2\theta - 3 = 0$$

giving your answers as exact multiples of π.

(5)

---

5. (*a*) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 10y = 27e^{-x}$$

(6)

(*b*) Find the particular solution that satisfies $y = 0$ and $\dfrac{dy}{dx} = 0$ when $x = 0$.

(6)

---

6. The transformation T from the z-plane, where $z = x + iy$, to the w-plane, where $w = u + iv$, is given by

$$w = \frac{4(1-i)z - 8i}{2(-1+i)z - i}, \qquad z \neq \frac{1}{4} - \frac{1}{4}i$$

The transformation T maps the points on the line l with equation $y = x$ in the z-plane to a circle C in the w-plane.

(*a*) Show that

$$w = \frac{ax^2 + bxi + c}{16x^2 + 1}$$

where a, b and c are real constants to be found.

(6)

(*b*) Hence show that the circle C has equation

$$(u - 3)^2 + v^2 = k^2$$

where k is a constant to be found.

(4)

---

7. (*a*) Show that the substitution $v = y^{-3}$ transforms the differential equation

$$x\frac{dy}{dx} + y = 2x^4y^4 \qquad \text{(I)}$$

into the differential equation

$$\frac{dv}{dx} - \frac{3v}{x} = -6x^3 \qquad \text{(II)}$$

(5)

(*b*) By solving differential equation (II), find a general solution of differential equation (I) in the form $y^3 = f(x)$.

(6)

8.

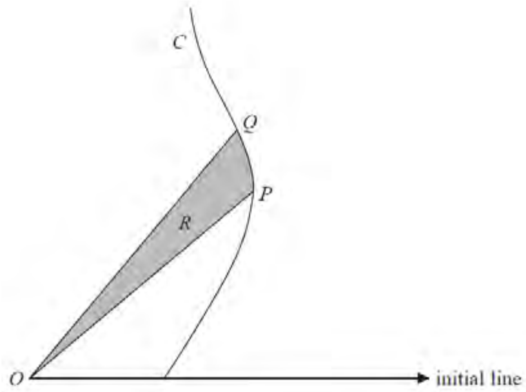

Figure 1

Figure 1 shows a sketch of part of the curve C with polar equation

$$r = 1 + \tan\theta, \qquad 0 \leq \theta < \frac{\pi}{2}$$

The tangent to the curve C at the point P is perpendicular to the initial line.

(*a*) Find the polar coordinates of the point P.

(5)

The point Q lies on the curve C, where $\theta = \frac{\pi}{3}$.

The shaded region R is bounded by OP, OQ and the curve C, as shown in Figure 1.

(*b*) Find the exact area of R, giving your answer in the form

$$\frac{1}{2}\left(\ln p + \sqrt{q} + r\right)$$

where p, q and r are integers to be found.

(7)

TOTAL FOR PAPER: 75 MARKS

END